Minimum jerk traction of a wheeled mobile robot passing over a hump

A. Ayalew, E. Lai
Dept. of Mech. & Manuf. Eng.
The Nottingham Trent University
Nottingham NG1 4BU
assefa.ayalew@ntu.ac.uk , e.lai@ntu.ac.uk

S. O. Oyadiji
School of Engineering
The University of Manchester
Manchester M13 9PL
sooyadiji@fs1.eng.man.ac.uk

Abstract

In this work a new technique for minimising jerk during longitudinal manoeuvre of a wheeled mobile robot traversing fixed road obstacles, which have positive gradient geometry, is presented. The dynamics of the robot with respect to the longitudinal, pitching and vertical motions of the platform are considered. The work has demonstrated that by controlling the drive torque input to the wheels, the platform can move over a positive-gradient hump with minimal jerk. This helps to minimise the possibility of wheel-ground contact loss, thus ensuring the stability of payload on wheeled mobile robots which are often used for transporting materials over uneven factory shop floors.

1. Introduction

There has been progress in the control of wheeled mobile robots, which are manoeuvred, on smooth terrain. Both kinematic- and dynamic-based control methodologies have been implemented for ideal indoor applications with no road obstacles [1][2][3]. But in a real industrial workspace these robots will encounter various obstacles like humps and cracks, which can bring about unwanted dynamic effects. Loss of control forces due to wheel-ground contact loss is one of such unwanted effects. This is because a wheel that has lost its contact with the ground can not generate either steering or traction forces. Consequently, it will be difficult to control the vehicle smoothly with a possible adverse impact on the stability of the payload on the platform.

Some consideration has been given to vehicle behaviour while moving on uneven terrain with respect to path planning. Siméon [4] considered the stability of a mobile platform as deemed to be satisfactory, when the line of action of the force of gravity on the vehicle intersected the polygon prescribed by the wheel contact points. In the path planning criteria used by Hait et al. [5], road paths that were likely to lead to wheel-terrain collision were included in the analysis but without considering the consequences. Ben Amar et al. [6] proposed a traction-force prediction scheme for vehicles travelling on uneven terrain based on the criteria that the wheels do not slip.

Proc. TIMR 99 “Towards Intelligent Mobile Robots”, Bristol 1999.
Technical Report Series, Department of Computer Science, Manchester University, ISSN 1361-6161.
Report number UMCS-99-3-1. http://www.cs.man.ac.uk/cstechrep/titles99.html

A highly simplified dynamics model has been presented by Shiller et al. [7] for vehicles travelling on uneven terrain. In this model a vehicle is reduced to a point mass so that the inertia effects can be ignored. No previous work, to the best of our knowledge, has addressed the issue of dynamic stability, in relation to the maintenance of wheel-ground contact and payload stability, for rigid mobile platforms travelling over road humps.

The validity of the constraint governing the wheel-ground contact is dependent on the acceleration at a contact point [8][9]. For a wheel to remain in contact with the road surface the normal component acceleration at the point of contact must be zero. If this component of acceleration is greater than zero at some time *t*, then contact is lost immediately after that instant. The acceleration can never be less than zero for rigid bodies, as this would imply that the wheel has penetrated the road surface. It is assumed that both the wheel and the road surface are rigid.

For contact to be maintained continuously, not only that the normal contact point accelerations must be zero but also their rate of change (jerk) must also be zero.
The normal accelerations are functions of the geometry of the road surface, and the angular and translational velocities and accelerations of the platform. By considering the case of a three-degree-of-freedom motion of a platform traversing a laterally symmetrical road obstacle (Figure 1), the present study attempts to address the issue pertaining to the maintenance of wheel-ground contact. The analysis assumes that the road obstacles have relatively small dimensions compared to the size of the vehicle (length of the vehicle and height of its centre of mass).

2. Modelling

Consider a rigid four-wheeled platform moving over a positive gradient hump. The wheels (front or rear) are subjected to a three degrees of freedom motion, namely, pitching motion about the lateral axis, longitudinal and vertical translational motion (Figure 1).

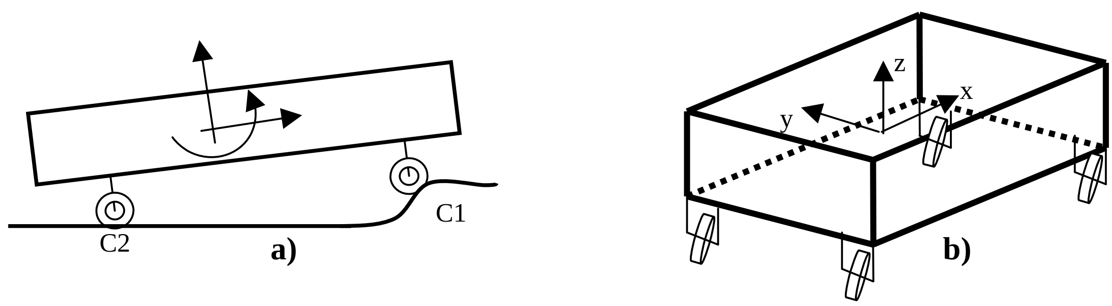

Figure 1. a) The three active degrees of freedom when passing over a hump.
b) Co-ordinate system definition.

The Newton's equations of motion, based on a co-ordinate system *(x-y-z)* attached to the vehicle's centre-of-mass can be written as:

$$\sum F_x = m(\dot{v}_x + v_z\omega_y)\,,\ \sum F_z = m(\dot{v}_z - v_x\omega_y)\,,\ \sum M_y = I_y\alpha_y \qquad \textbf{(1)}$$

where m is the mass and I_y is the pitch moment of inertia, v_x and v_z are the longitudinal and the vertical velocities, respectively, ω_y is the pitch angular velocity, and α_y is the pitch angular acceleration of the platform.

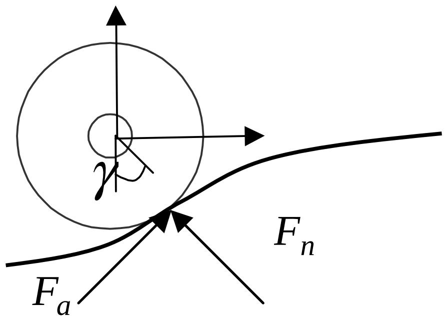

Figure 2. The geometry of wheel forces.

The wheel forces at the wheel k can be written as (Figure 2):

$$\boldsymbol{F_{wk}} = \boldsymbol{A_k} F_{ak} + \boldsymbol{N}_k F_{nk} \qquad \textbf{(2)}$$

F_a and F_n are the traction and normal reaction forces. $\boldsymbol{A}_k$ and $\boldsymbol{N}_k$ are given by:

$$\boldsymbol{A}_k = \begin{bmatrix} \cos(\gamma_k)\cos(\delta_k) \\ \cos(\gamma_k)\sin(\delta_k) \\ \sin(\gamma_k) \end{bmatrix},\ \boldsymbol{N}_k = \begin{bmatrix} -\mu_{rk}\cos(\gamma_k)\cos(\delta_k) + \mu_{lk}\sin(\delta_k) - \sin(\gamma_k)\cos(\delta_k) \\ -\mu_{rk}\cos(\gamma_k)\sin(\delta_k) - \mu_{lk}\cos(\delta_k) - \sin(\gamma_k)\sin(\delta_k) \\ -\mu_{rk}\sin(\gamma_k) + \cos(\gamma_k) \end{bmatrix}$$

μ_l is the lateral friction coefficient and μ_r is the rolling resistance coefficient, δ is the steering angle of the wheel, and γ is the angle which specifies the direction of the normal reaction force, it is also the gradient of the profile of the road surface at the point of contact. Although the contact forces (Equation 2) have been written for the most general case of a steered wheel, the present study concentrates on the special case of the longitudinal motion of a four-wheel vehicle. Hence the lateral friction forces are neglected with $\delta_k = 0$.

The acceleration at an arbitrary point *p* on the platform is written as:

$$\boldsymbol{a_p} = \boldsymbol{a_c} + \boldsymbol{\omega}\times\boldsymbol{\omega}\times\boldsymbol{r_{p/c}} + \boldsymbol{\alpha}\times\boldsymbol{r_{p/c}} \qquad \textbf{(3)}$$

where $\boldsymbol{r_{p/c}}$ is the position of the point *p* with respect to the platform's centre of mass *c*.

Two other equations are needed for solving the equations of motions for the unknown accelerations and wheel reactions forces. These can be derived from the normal acceleration components of the rear and front wheel contact points, C1 and C2 (Figure 1). Let $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ be the normal vectors to the road surface at the front and rear wheel contact points, respectively, and let $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ be the corresponding accelerations.

Thus,

$$\boldsymbol{a}_1 \bullet \boldsymbol{n}_1 = 0 \text{ and } \boldsymbol{a}_2 \bullet \boldsymbol{n}_2 = 0 \tag{4}$$

The equations of motion (Equation 1) can be solved to find the normal wheel reaction forces, the platform's centre of mass acceleration $\boldsymbol{a}_c$ and its pitch angular acceleration α_y after substituting for the wheel forces given in Equations 2 and using the Equations 3 and 4.

The rate of change of acceleration, that is jerk, at the point *p* on the platform, $\boldsymbol{Je}_p$, is given by the differential of the right hand side of Equation 3 with respect to time. It is evident that this quantity is the sum of the jerk of the centre of mass ($\boldsymbol{Je}_c = \frac{d}{dt}\boldsymbol{a}_c$), the rate of change of the centrifugal acceleration term, $\frac{d}{dt}(\boldsymbol{\omega}\times\boldsymbol{\omega}\times\boldsymbol{r}_{p/c})$, and the rate of change of the tangential acceleration term, $\frac{d}{dt}(\boldsymbol{\alpha}\times\boldsymbol{r}_{p/c})$.

The centrifugal acceleration term contributes a jerk, which is proportional to the third power of the pitch angular velocity and another term, which is proportional to the product of the pitch angular velocity and the pitch angular acceleration. Since the angular velocity and angular acceleration are very small in so far as small size road obstacles are concerned, the contribution of the centrifugal acceleration term can be neglected. Hence the total jerk can be minimised by minimising the translational jerk at the centre of mass and the pitch angular jerk of the platform. Since we are dealing with acceleration with respect to a non-inertial-frame-of reference *x*-*y*-*z*, the jerk we are dealing with is one sensed by an observer moving with the vehicle.

The components of $\boldsymbol{a}_c$ are functions of the gradient of the road profile γ, the longitudinal velocity v_x, and the vertical velocity v_z hence the differentiation of acceleration $\boldsymbol{a}_c$ with respect to time is carried out in the following manner.

$$\boldsymbol{Je}_c = \frac{d\boldsymbol{a}_c}{dt} = \frac{\partial \boldsymbol{a}_c}{\partial \gamma}\frac{d\gamma}{dt} + \frac{\partial \boldsymbol{a}_c}{\partial v_x}\frac{dv_x}{dt} + \frac{\partial \boldsymbol{a}_c}{\partial v_z}\frac{dv_z}{dt} \tag{5}$$

Similarly, for the pitching motion, the jerk is calculated as:

$$Je_{\alpha y} = \frac{d\alpha_y}{dt} = \frac{\partial \alpha_y}{\partial \gamma}\frac{d\gamma}{dt} + \frac{\partial \alpha_y}{\partial v_x}\frac{dv_x}{dt} + \frac{\partial \alpha_y}{\partial v_z}\frac{dv_z}{dt} \tag{6}$$

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dx}\frac{dx}{dt} = \frac{d\gamma}{dx}v_x \tag{7}$$

The term $\frac{d\gamma}{dx}$ is the curvature of the road profile at the point of contact and can be estimated for a continuous, smooth profile by differentiating the record of γ with respect to longitudinal displacement *x*.

3. Theoretical Results

The jerks of the translational motions of the centre of mass and the pitch angular motion of the platform, as felt by an observer travelling with the vehicle, are mainly functions of the geometry of the profile and the magnitude of the actuating forces. By solving the relevant governing equations, the relationship between the traction force requirement and the road surface profile (in terms of γ) can be established. For example, consider a mobile robot of the following geometric dimensions, and inertia and kinematic parameters:

Distance between front and rear axle, L=2.2 m;
Distance between front axle and centre of mass, Lr=1.0 m;
Height of centre of mass, H=0.42 m;
Total mass of vehicle, *m*=50 kg;
Pitch moment of inertia, *Iy*=80 kg-m^2;
Longitudinal velocity of vehicle, v_x=2.5 m/s.

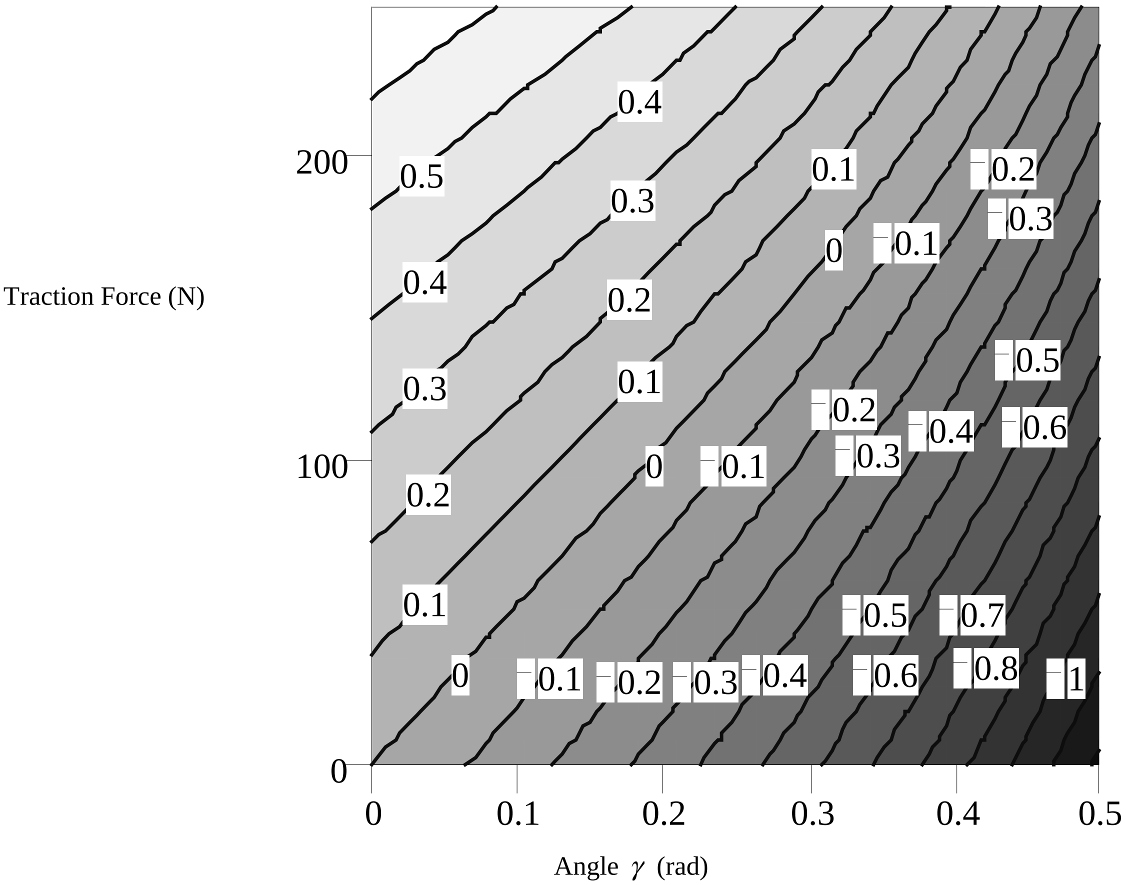

Figure 3-a) Contour plot of the longitudinal jerk.

Using the above information, the jerks (Equation 5-6) and accelerations (Equation 1) become functions of two variables, namely the wheel traction force (F_t) and the profile gradient (γ). The functional relationship between jerk/acceleration, traction force (F_t) and profile gradient (γ) can be represented graphically in the form of contour plots. In this example we have used a traction force ranging from zero to 250 N and profile gradient ranging from zero (corresponding to flat surface) to 0.5 radians ($\approx 30^o$ inclination from the horizontal).

For each pair of values of traction force and profile gradient, that is (F_t, γ), the corresponding values of jerk and acceleration were computed using Equations (1), (5) and (6). The results are depicted as contour plots of longitudinal jerk and longitudinal acceleration in Figures 3-a and 3-b respectively. In Figure 3-a, the family of curves denote lines of constant jerk while the numerical values associated with each curve indicate the numeric values of the jerk. Similarly, in Figure 3-b, the family of curves denote lines of constant acceleration while the numerical values associated with each curve indicate the numeric values of the acceleration.

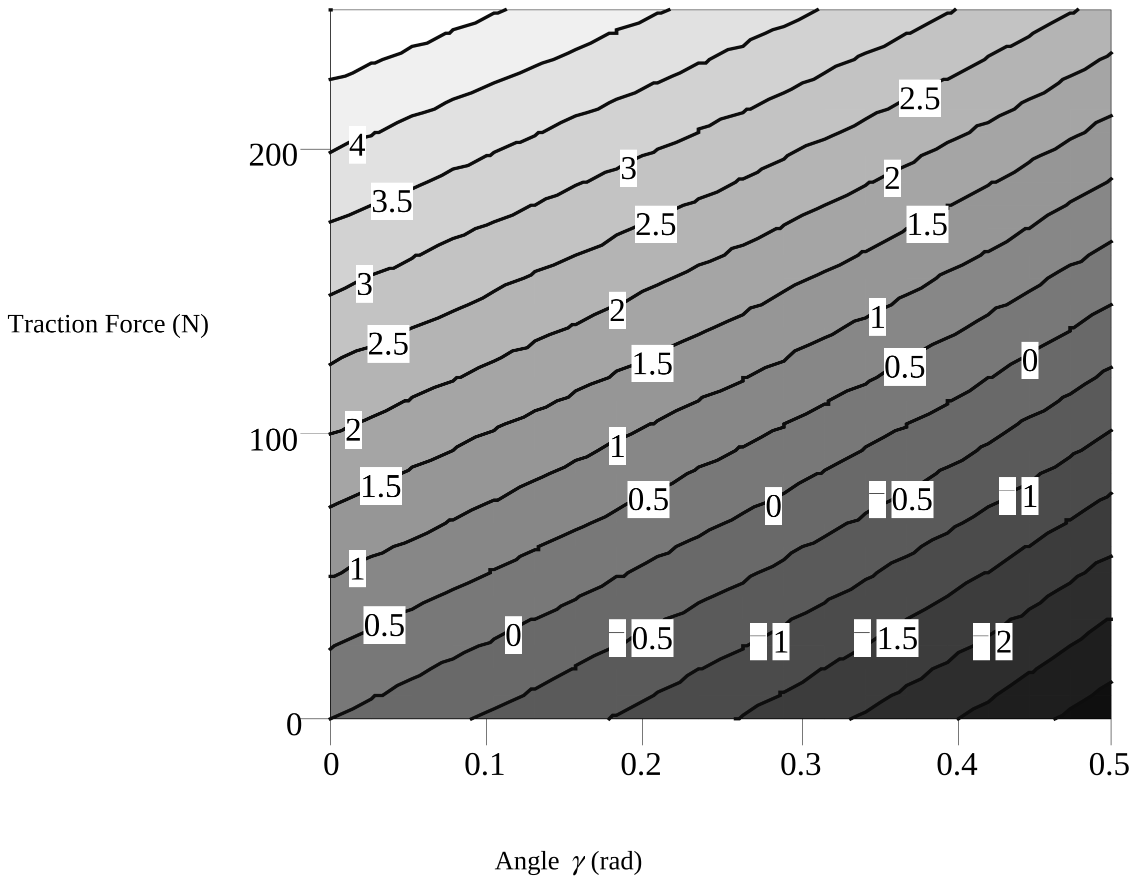

Figure 3-b) Contour plot of longitudinal acceleration

The zero contour line in Figure 3-a represents the conditions required for a theoretical jerk-free motion, if a proper actuating force is selected as a function of the geometry of the profile. It has also been that an identical zero-jerk contour line is present in both components of the translational motions (vertical and longitudinal), as well as the pitch angular rotation. Hence, it can be concluded that for a given profile gradient, an appropriate traction force can be determined and applied to the wheels in order to ensure that the jerk in all motions is instantaneously zero. It should be noted that the longitudinal jerk and the longitudinal acceleration can not be simultaneously zero. The desired state is for the jerk to remain at zero throughout the duration of the motion of the mobile robot. This requires that the robot's longitudinal acceleration be positive.

4. Conclusions

A technique has been presented for the minimisation of jerk during longitudinal manoeuvre of a wheeled mobile robot traversing fixed road obstacles, which have positive gradient geometry. This can be accomplished by controlling the traction forces applied at the wheels. The traction force requirement for a given application can be determined from knowledge of the geometry of the road profile and vehicle dimensions.

5. References

1. Z. Deng and M. Brady, "Dynamics and Control of a Wheeled Robot," *Proceedings of the American Control Conference.*
2. *M. G. Mehrabi and A. Hemami,* "Dynamic Modelling of Wheeled Mobile Robots. Theory and Experiment," *Second IEEE Conference on Control Applications. September 1993.*
3. J.C. Alexander and J.H. Maddocks, "On The Kinematics of Wheeled Mobile Robots," *The International Journal of Robotics Research. Vol. 8, No. 5. Oct 1989.*
4. T. Simeon, "Motion Planning for a Non-holonomic Mobile Robot on 3-dimensional Terrain," *IEEE/RSJ International Workshop on Intelligent Robots and Systems, IROS'91.*
5. A. Hait and T. Simeon, "Motion Planning on Rough Terrain for an Articulated Vehicle in Presence of Uncertainities," *IEEE International Conference on Intelligent Robots and Systems, 1996.*
6. F. Ben Amar, Ph. Bidaud and F. Ben Ouezdou, "On Modelling and Motion planning of Planetary Vehicles," *Proceedings of the 1993 IEEE/RSJ International Conference on Intelligent Robots and Systems.*
7. R. Featherstone. Robot Dynamics Algorithms, *Kluwer Academic Publishers. 1987.*
8. D. Baraff, "Issues in Computing Contact Forces for Non-penetrating Rigid Bodies," *Algorithmica, 1993.*
9. Z. Shiller and Y. Gwo, "Dynamic Motion Planning of Autonomous Vehicles," *IEEE Transactions on Robotics and Automation. 1991*