CS2411 - Study Problems 1

These problems are to review basic probability theory and to move you along to Bayesian classification. The example class is an opportunity to get help or hints. Problems marked with a # are harder or more mathematical. It is most important that you be able to do the unmarked questions.

Simple Probability Calculus

1. In craps (a game played with a pair of dice) you lose if you roll a 2 or an 11 on the first roll. What is the probability of losing on the first roll?

2. Two brothers skip school one morning and give the teacher the false excuse that they had a flat tire. The teacher separates them and asks them to each write down which tire was flat. What is the probability that they both chose the same tire (assuming that they both guess)?

3. What is the probability of winning the jackpot on the National Lottery? (To win the jackpot, one must match six different numbers between 1 and 49 in any order).

4. Suppose in a given week the expected payoff in the National Lottery is 8 million pounds (reduced slightly from the jackpot due to possibility of shared prizes). Suppose a person buys a ticket for one pound. Using probability as a model of belief, what can you say about that person's *belief* that they will win as compared to the actual odds of winning? (Assume that the person behaves rationally and plays the lottery solely to win. Are there other assumptions required to answer this question?)

5. Suppose it is the case that 19% of females smoke and 35% have blond hair. What is the probability that a randomly selected woman is both a smoker and blond? What is the probability that she is either?

6. Make a histogram of the data in section 5.3 of the probability notes. What is your estimate of the probability of the value 35?

Bayes Rule

1. The English Speaking Traveller (from Pao). Suppose you are visiting a country in which:

   - One out of 10 natives speaks English
   - One out of five persons is a tourist
   - One out of two tourists speak English.

   You meet another person and converse in English. Would you deduce that they are native? (*Use Bayes rule to compute the probability that they are native.*)

2. In a city of a million inhabitants, a murder is committed. You are accused of the crime, although you are innocent. Some hair is found at the scene which the genetic fingerprinting expert claims matches your DNA, although you have never been there. During the trial, the prosecutor asks this expert, "what is the possibility of getting this same DNA signal from someone other than the defendant?" The expert replies, "the probability of getting this DNA signal given that the hair is not from the defendant's head is one in a

million." You turn to your lawyer who makes the motion of a knife across the throat with his finger. Express the information provided by the expert as a conditional probability. What is the conditional probability that you committed the crime, given his evidence? Use Bayes rule to compute it and get yourself acquitted.

From Probability to Statistics

1. Suppose there is a process which produces identical computer chips and it is your job to determine the reliability of the process. You assume that the process produces a bad chip with probability p.

    (a) Suppose you know that p is 0.1. What is the probability that in a selected 10 chips, no bad chips will be found?

    (b) Suppose you test 10 chips and 5 are found to be bad. What is your estimate of p? How much would you expect the actual value of p to vary from your estimate? What if you had tested 100 chips and found 50 to be bad?

    (c) # Suppose you test 10 chips and none are found to be bad. What can you conclude about p? In particular, how big can p so that this result is within a 95% interval?