QUANTITATIVE ANALYSIS OF ROBOT-ENVIRONMENT INTERACTION — ON THE DIFFERENCE BETWEEN SIMULATIONS AND THE REAL THING*

Ulrich Nehmzow
Department of Computer Science
The University of Manchester
Manchester M13 9PL
United Kingdom
ulrich@cs.man.ac.uk

Abstract

This paper presents a quantitative analysis of trajectories of mobile robots or their computer simulations, based on the Error Growth Factor *(EGF), an approximation of the Lyapunov exponent.*

Using the EGF, it can be shown that deterministic chaos can be present in the behaviour of a mobile robot interacting with its environment, and that there is a substantial difference between physical mobile robots and their generic computer models.

1 Introduction

1.1 Motivation

A mobile robot operating in and interacting with its environment essentially performs a complex function that is governed by sensory perception, actuator performance and environmental factors. Few attempts have been made so far to quantify this function, or to characterise robot-environment in any other way but qualitatively.

In this paper, we present a quantitative measure of robot-environment interaction based on an analysis of the robot's trajectory over time. The Error Growth Factor (*EGF*), a measure that estimates how sensitive the robot's trajectory is to changes of initial conditions, i.e. how much two trajectories starting from marginally different starting locations would diverge over time, is used in the experiments reported here to assess whether a commercially available simulation model of the robot we used behaves in fundamentally the same way as the robot, or not.

There are may conceivable applications of such quantitative robot performance measures, including determining the influence of individual control parameters upon robot performance, the characterisation of environments or robots, the comparison of results obtained by different research groups, or even the development of a general theory of robot-environment interaction (see [3] for a discussion).

1.2 Related Work

Quantitative assessment of mobile robot behaviour is still not a common practice in mobile robotics research, partly due to the variety of environments, platforms and control programs used. However, some proposals for quantitative performance measures have been made, and applied to mobile robotics research.

Smithers [16] early on stimulated the discussion, proposing dynamical systems approaches to understand robot-environment interaction. This work is related to that of Schöner and co-workers [14, 15], who also apply dynamical systems theory to the analysis of robot behaviour.

Statistical methods have successfully been applied to compare for instance the behaviour of two performance strategies. Nehmzow and Nehmzow & Duckett [11, 6] present statistical methods to assess the performance of robot localisation systems.

Finally, some work has been done to quantify the difference between robots and their computer models. Gat [8] presents such a comparison between a mobile robot and its simulation, using elapsed time to reach a goal and distance travelled as performance metrics. Lee [10] discusses how to build a faithful computer model, based on real-robot data.

*Proc. Eurobot 2001

2 The Error Growth Factor

One of the characteristics of deterministic chaos is *sensitivity to initial conditions*. If deterministic chaos is present, two trajectories starting from two arbitrarily close starting positions will diverge eventually. On the other hand, if deterministic chaos is not present, trajectories starting from different starting locations will converge upon one trajectory eventually — the attractor.

A robot's behaviour in its environment is the result of many factors, such as the robot's properties, the world's properties and properties of the controlling program. The robot can be viewed as a kind of analog computer which takes those properties as input, and "computes" behaviour as an output. For the purpose of this paper, we consider this output, the behaviour, to be expressed by the trajectory taken by the robot.

If the resulting behaviour proved to be non-chaotic, this would mean that two trajectories that started from slightly different starting points would converge into one. Likewise, if chaotic behaviour was observed, it would mean that those two trajectories would diverge more and more as the robot moves.

It is possible to measure the rate of convergence or divergence. One way to characterise the divergence or converge of a trajectory is to compute the (first) Lyapunov exponent λ, as defined by equation 1 [13, p. 519].

$$\lambda(x_0) = \lim_{m\to\infty} \sum_{k=1}^{m} \ln |f'(x_{k-1})|, \qquad (1)$$

where $f(x)$ is a smooth transformation function, $f'(x)$ its derivative and x_0 is the initial point.

In the case of analysing robot behaviour, the transformation function $f(x)$ is unknown and the Lyapunov exponent must be estimated in some other way. In the experiments reported here, we have estimated the Lyapunov exponent from the robot's logged trajectory, analysing x and y component individually. The fundamental principle, detailed below, is to find two points $x(t_1)$ and $x(t_2)$ that lie very close to each other, and to estimate the Lyapunov exponent using equation 2.

$$\lambda \approx \ln |\frac{x(t_2) - x(t_2+1)}{x(t_1) - x(t_1+1)}|, \qquad (2)$$

Equation 2 is an estimate of the precise definition of the Lyapunov exponent λ given in equation 1, essentially approximating the derivative in equation 1.

The practical implementation, the calibration of the method and the application to a comparison between a robot and its simulation are discussed in the following sections.

2.1 Definition of the EGF

There are some practical problems associated with estimating the Lyapunov exponent from a time series, and established methods (eg. [17]) are often elaborate. For the initial investigation presented here we estimate the Lyapunov exponent by the method outlined below.

The exponential Error Growth Factor (EGF) of a time series $z(t)$ is an approximation of the first Lyapunov exponent [13], and defined by equation 3 (see also figure 1).

$$EGF = \ln(\frac{1}{j} \sum_{p} \sum_{k=0}^{n} \frac{|z(p+k+1) - z(p'+k+1)|}{|z(p+k) - z(p'+k)|}), \qquad (3)$$

with $p' = p + \Delta$ such that equation 4 is fulfilled, and $\Delta > n$.

$$d_{min} < |z(p) - z(p')| < d_{max}. \qquad (4)$$

j is the number of $z(p)/z(p')$ pairs can be found in the time series that meet the condition of equation 4, and d_{min}, d_{max} and n are parameters that need to be predefined by the user (how this is done is discussed in subsection 2.3).

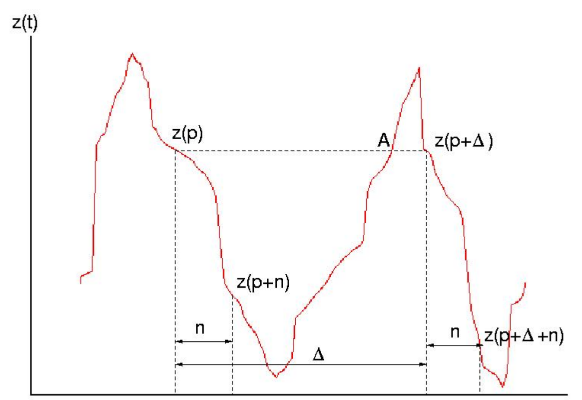

Figure 1: Computation of the Error Growth Factor (EGF) from a time series $z(t)$.

This method essentially finds two points $z(p)$ and $z(p')$ in the time series that are at very similar, but not identical points of the time series (minimum and maximum difference are defined by equation 4), and uses the difference between $z(p+n)$ and $z(p'+n)$ (with n being the number of timesteps taken between these measurements) to estimate whether there is a positive Lyapunov exponent (differences increasing over time) or a negative Lyapunov exponent (differences decreasing over time). How we determined factors like n, d_{min} and d_{max} is discussed in section 2.3.

2.2 Computation of the EGF

The EGF is computed from a sampled time series $z(t)$ — in the experiments presented here the x or y coordinate of the robot's trajectory, logged by the on-board odometry system. Examples of such time series are shown in figures 5 and 7.

The amplitude of the raw time-series is first reduced to the interval [0,1], to avoid errors in the computation of the *EGF* due to scale. For each data-point $z(t_i)$ of z the following procedure is then performed (see figure 1):

1. Find some $z(t_i + \Delta)$ with $d_{min} < |z(t_i) - z(t_i + \Delta)| < d_{max}$ with Δ being some number greater than n, ensuring that the sign of the first derivative (i.e. the slope) at points $z(t_i)$ and $z(t_i + \Delta)$ is the same (point A in figure 1 would not meet this criterion). $d_{min} = 10^{-5}$, $d_{max} = 10^{-2}$ and $n = 20$ for the results presented in this paper.

2. Compute $\sum_{k=0}^{n} \frac{|z(t_i+k+1)-z(t_i+\Delta+k+1)|}{|z(t_i+k)-z(t_i+\Delta+k)|}$

3. Repeat steps 1 and 2 for all remaining data-points of z up to $l-n-1$ (l is the total number of data-points in z).

4. Compute the *EGF* according to equation 3.

2.3 Calibration

To determine the parameters n, d_{min} and d_{max} the procedure discussed in section 2.2 was applied to the quadratic iterator z_q given in equation 5.

$$z_q(t) = a z_q(t-1)(1 - z_q(t-1)). \qquad (5)$$

The Lyapunov exponent for the quadratic iterator can be computed precisely [13, p. 518], which means that it can be used to calibrate the computation of the *EGF*.

For $a = 2.5$ the Lyapunov exponent is -0.69, the computed *EGF*, using the parameters given above, is also computed as -0.69. For $a = 4$ the Lyapunov exponent is 0.69, in this case the *EGF* is computed as 0.63, which is a tolerable error for the application discussed here.

Figure 2: The Nomad 200 mobile robot FortyTwo used in the experiments reported here.

3 Experimental Comparison between Simulated and Real Robots

3.1 Experimental Procedure

Three different control programs were executed both on Manchester's Nomad 200 mobile robot *FortyTwo* (see figure 2) and the Nomad 200 simulator supplied by the manufacturers of the robot [12].

The three control programs provided three fundamentally different kinds of robot-environment interaction, in order to investigate under what circumstances a difference between simulation and real robot could be detected (if any).

Control program 1 used no sensors whatsoever and simply made the robot/simulation move in a circle. Any difference between simulator and robot would in this case be due to actuator noise alone. The observed robot and simulation trajectories are shown in figure 3.

Control program 2 provided a loose coupling between perception and action, using a forcefield approach to obstacle avoidance. For most of the time the robot/simulation moved freely, but was gradually "pushed" sideways as the robot/simulation approached an obstacle. Figure 4 shows the robot and simulation trajectories observed, figure 5 the x-coordinate versus time of both robot and simulation.

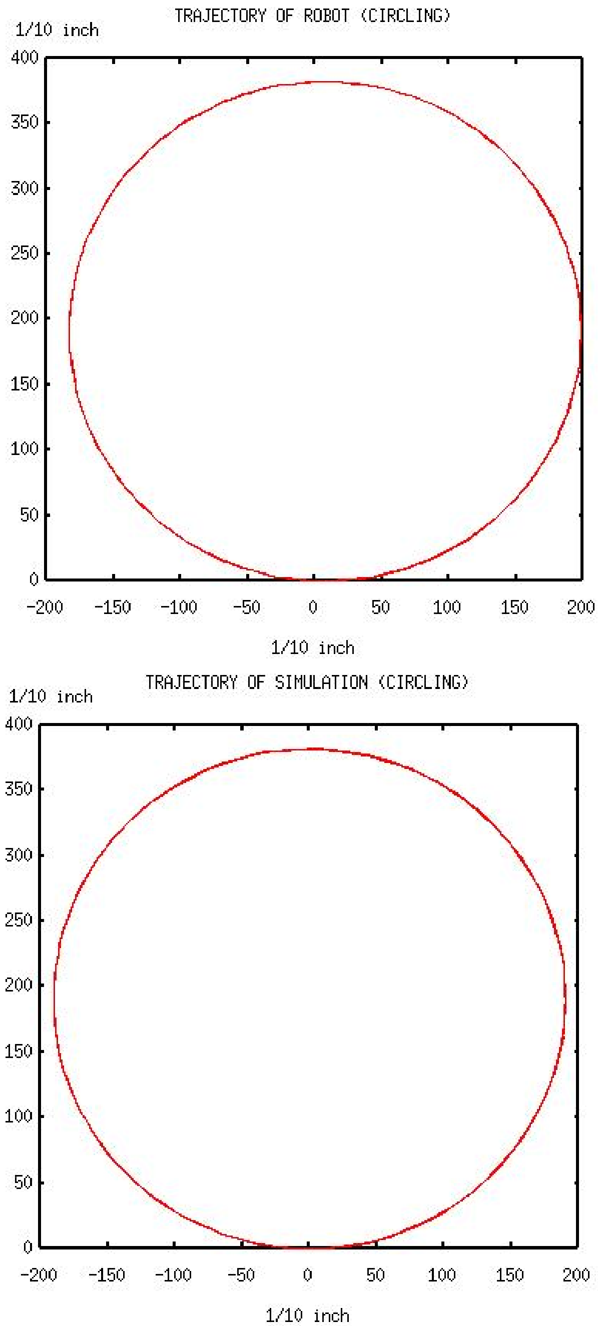

Figure 3: Robot (top) and simulation trajectory logged when executed the actuators-only "circle" control program.

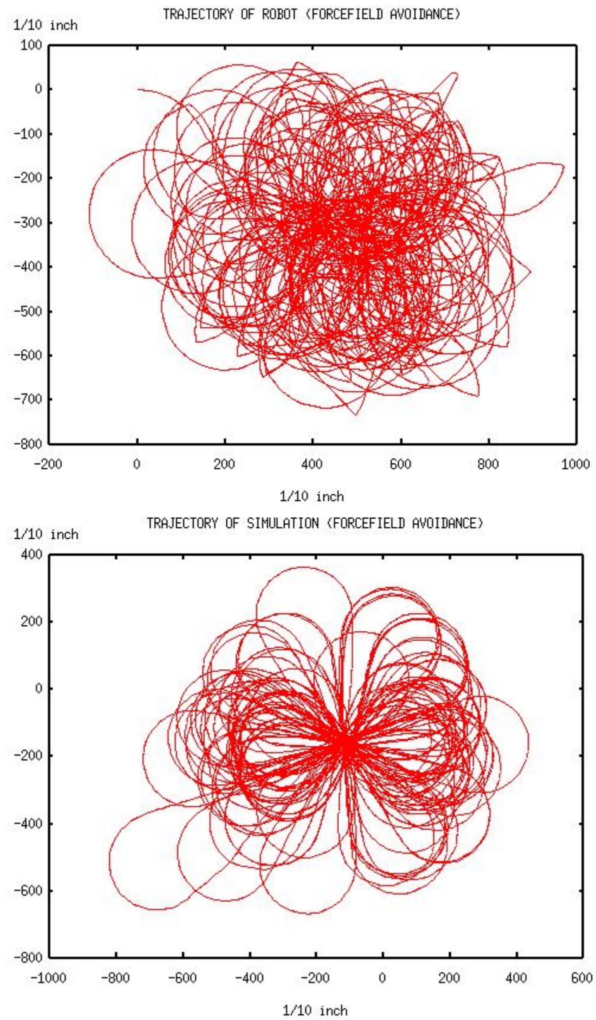

Figure 4: Robot (top) and simulation trajectory logged when executed the "forcefield obstacle avoidance" control program.

Finally, control program 3 provided tight coupling between perception and action by making the robot/simulation stay within close proximity to walls (wall following). Figure 6 shows the robot and simulation trajectories observed. Figures 7 shows the y-coordinate of these trajectories over time, which were used to compute the EGF.

3.2 Results

It is obvious from visual observation of the trajectories shown in figures 3, 4 and 6 that at least in the latter two simulation and robot trajectory differ considerably. The question is whether this difference can be quantified.

The computed EGF for the three control programs, for both simulation and real robot, are given

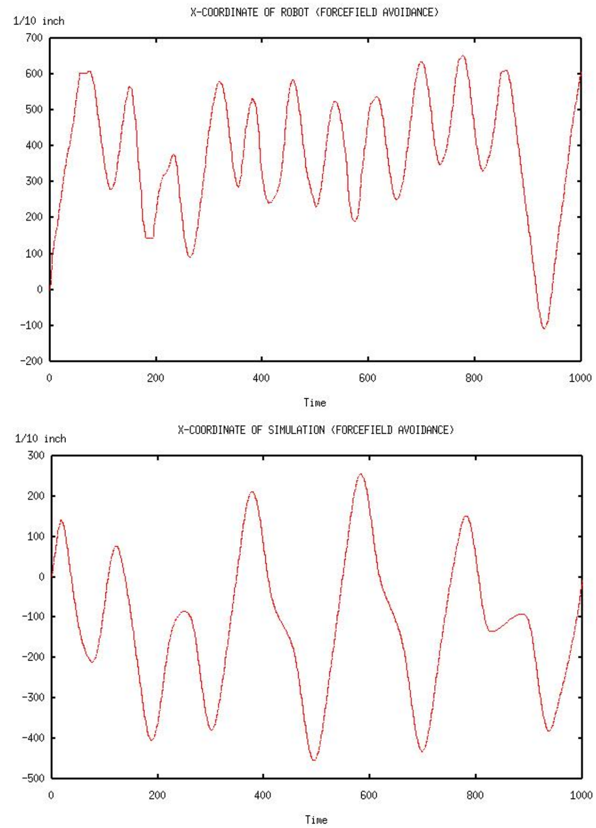

Figure 5: X-coordinate of robot (top) and simulation, performing forcefield obstacle avoidance.

in table 1.

The results obtained with the "circle" actuator-only control program are surprising, in that the simulation has a higher EGF than the actual robot. This result requires further investigation — our current conjecture is that the actual robot actuators are more precise than the noisy simulation (in other words: the noise level selected in the simulator is too high to model the robot's actuators accurately). It should be borne in mind that the simulator isn't designed to model actuators only, and that the noise levels modelled aim to reflect the noise present when actuator commands are influenced by perception.

As soon as sensor-motor tasks are executed, however, it becomes apparent that the degree of chaotic behaviour present in the robot is higher than in the simulation. Comparing the results in the case of forcefield obstacle avoidance — which only uses loose sensor-motor couplings — it can be seen that even in this case the EGF of the robot is higher than that of the simulation.

Even more pronounced are the differences in EGF obtained using the wall following control program. This program maintains a tight coupling between sensory perception and motor response: in order to stay within close proximity to the wall, constant adjustments to the robot's trajectory are necessary. Any noise-induced chaotic behaviour of sensors or actuators will be most pronounced in this case.

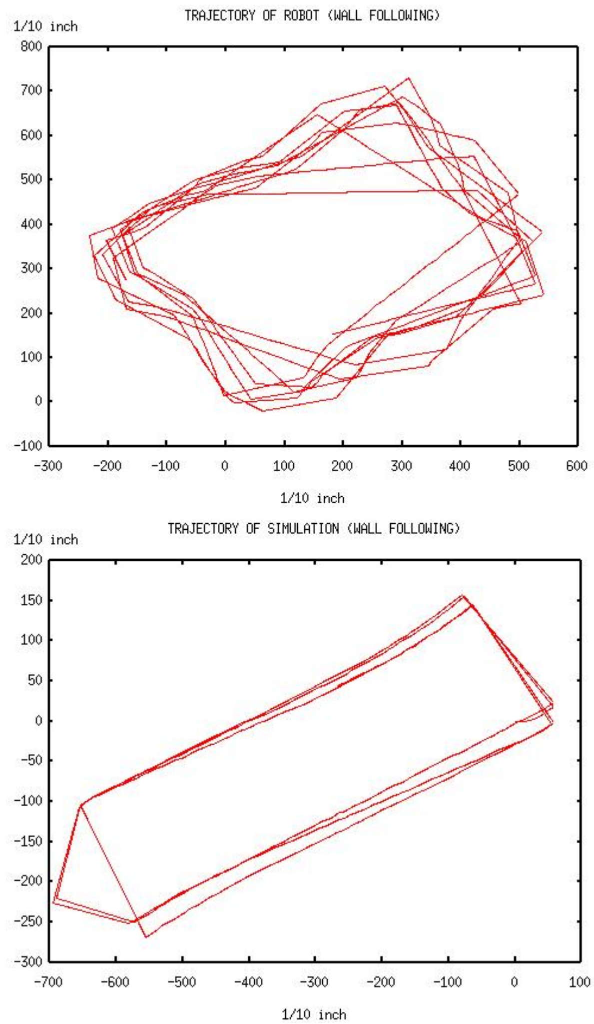

Figure 6: Robot (top) and simulation trajectory logged when executed the "wall following" control program.

As the wall following program had been designed specifically with the objective to produce repeatable and predictable trajectories, it is interesting to see that in the wall following behaviour, the robot's EGF indicates a significantly larger error growth (chaotic behaviour) than the simulation. Especially comparing the EGF obtained by using the x coordinate, the robot's EGF is more than twice as large as the simulation's. This result indicates that for programs showing a tight coupling between perception and action — this is the majority of "realis-

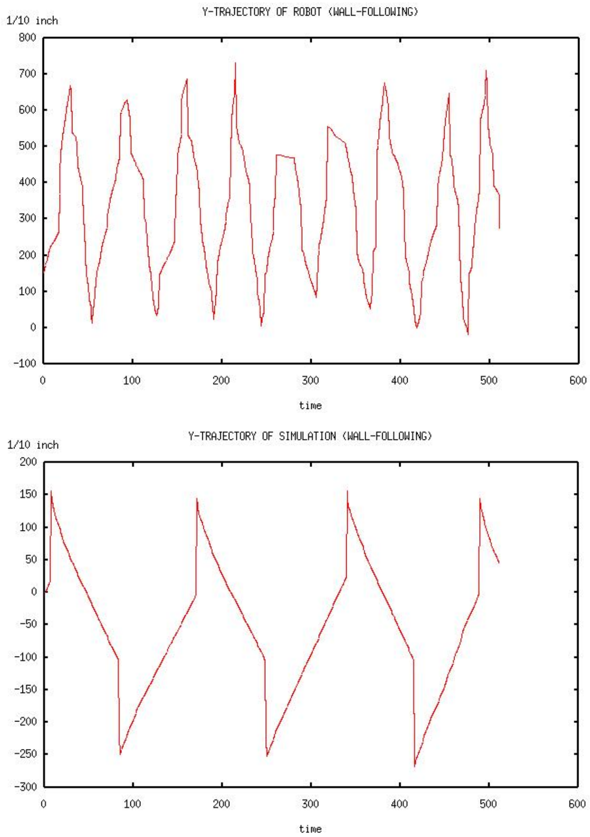

Figure 7: y-coordinate versus time of robot (top) and simulation, executing the wall-following program.

tic" robot control programs — a fundamental difference exists between a physical mobile robot and its simulation. The robot's EGF indicates that deterministic chaos underlies the robot's behaviour, which raises the question whether faithful modelling of robot-environment interaction is at all possible. The preliminary results presented here suggest that this is *not* the case.

4 Conclusion

4.1 Discussion

We believe that the method presented here of estimating the Lyapunov exponent is suitable for quantitative analysis of robot behaviour. However, there are two important aspects that have to be considered.

First, estimating the Lyapunov exponent from a time series typically requires large amounts of data to be accurate [7, 17, 4]. 1000 data-points is usually not sufficient, so in order to increase confidence in our results longer time series should, if possible, be obtained.

	SimX	SimY	RobX	RobY
C 400	0.32	0.28	0.20	0.21
F400	0.21	0.21	0.30	0.30
F1000	0.18	0.20	0.28	0.30
F10000	0.17	0.20	0.28	0.29
WF400	0.31	0.62	0.70	0.58
WF1000	0.33	0.56	0.69	0.68
WF10000	0.35	0.57	n/a	n/a

Table 1: Error Growth Factors computed for simulation and robot, for the "circle (C)", "forcefield avoidance (F)" and "wall follow (WF)" control programs, using the x-component and the y-component of trajectories to compute the EGF. Numbers following "C", "F" and "WF" indicate the number of data points in the time series used. "x" and "y" indicate whether x- or y-coordinate versus time were used to compute the EGF.

Nevertheless, the results shown in table 1 indicate little difference for 400, 1000 and 10000 data-points, and we therefore argue that while the numbers presented in table 1 may not represent the *exact* Lyapunov exponent, they give a reasonably accurate approximation of it.

For future work, we intend to obtain longer time series either by using external logging equipment (e.g. overhead cameras), or by constructing nonlinear models based on relatively short time-series (a few hundred data-points), and to use these to estimate the Lyapunov exponent. NARMAX models have successfully been used for this [1].

Secondly, the time series we used were obtained using the logged odometry data of the robot. As robot odometry is subject to drift error, this means that any growth in error determined describes the entire system of robot, environment *and* odometry system. In other words, the odometry system itself contributes towards the EGF. For the purpose of this paper this is no shortcoming of the experimental approach. We were interested to detect and quantify any existing difference between a physical mobile robot and its computer model. Odometry error is one aspect of the physical mobile robot, and is also modelled in the simulator. The proposed method can therefore still detect differences between robot and simulation.

The results obtained show that there is a distinct difference between the behaviour of our Nomad 200 mobile robot and its computer model. For "realistic" control programs with a tight coupling between sensing and perception the robot's EGF is considerably larger than the simulation's, indicating the presence of deterministic chaos. Whether chaotic systems can be simulated using generic mathematical models is debatable.

Particularly interesting are the results obtained using the wall-following control program. This program was specifically designed to result in identical, stable trajectories, irrespective of the robot's (or the simulation's) starting position. It was envisaged that the nearby wall would serve as an attractor, leading the robot or the simulation to stable paths.

The results, however, showed that there was a considerable difference between the robot and the simulation. While both showed a positive EGF, indicating the presence of deterministic chaos, the EGF in the robot was considerably larger. The fact that a program that was designed with repeatable and stable trajectories in mind produces such radically different results between robot and simulation indicates that a simple generic model of robot sensors and actuators is insufficient to model a mobile robot faithfully. This observation agrees with results we obtained earlier [9].

4.2 Outlook

To develop quantitative methods for the analysis of robot-environment interaction is, in our opinion, vitally important to advance the field of mobile robotics. Due to the lack of such tools, experimental results currently have to be presented in qualitative terms, and constitute not much more than existence proofs. This is useful, but mobile robotics has by now reached a stage where the move towards an exact science is possible.

In this paper we have shown how one such quantitative measure — the error growth factor, an estimate of the Lyapunov exponent — can be used to compare the actual behaviour of a mobile robot with predictions made by a computer model. The observation was that for all but the simplest of programs the simulator behaves fundamentally different to the real robot. Simulator predictions therefore have to be treated with caution.

This paper reports ongoing work. The next steps are to obtain longer trajectory logs by means of an overhead camera or other means of external observation. Regarding methods of analysis, lag diagrams, system identification [1, 2, 5] and related methods from chaos theory will be applied.

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